JMEPEG (1999) 8:75-82 ©ASM International

The Mathematical Modeling of Phase Transformation of Steel during Quenching

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(Submitted 15 December 1997; in revised form 22 June 1998)

In the heat treatment of steel, uneven cooling invariably introduces residual stresses in the workpiece. These residual stresses can combine with the thermomechanical stresses encountered in operation to cause premature fatigue failure of the material. A prediction of the residual and thermoelastoplastic stresses developed during heat treatment would be beneficial for component design.

In this article a numerical model is developed to predict the thermoelastoplastic and residual stresses during rapid cooling of a long solid cylinder. The total strains developed during cooling of the cylinder comprise elastic, thermal, and plastic strains and strains due to phase transformation. For plastic deformation an extension of Jiang's constitutive equations developed by Jahanian is adopted. The properties of the material are assumed to be temperature dependent and characterized by nonlinear strain hardening.

For phase transformation two parts are considered: nucleation according to Scheil's method and phase growth according to Johnson and Mehl's law. For martensitic transformation, a law established by Koisteinin and Marburger is used. Non-additivity of pearlitic and bainitic nucleation suggested by Manning and Lorig is taken into account by means of a correction factor to Scheil's summation of the transition from pearlitic to bainitic.

The effect of phase transformation and temperature dependence of material properties is investigated. It is shown that by neglecting the temperature dependency and phase transformation in numerical calculations, the results are underestimated. The numerical results are compared with the available experimental data in the literature, and good agreement is observed.

Keywords modeling, phase transformations, quenching, quenching steel

1. Introduction

The stability and dynamic behavior of mechanical structures can be altered by a change in the residual stress present in the material. The fatigue life can be increased by simply developing beneficial residual stresses in a mechanical component. With further insight into the stresses and strains developed during heat treatment, the level of residual stresses can be predicted. With this knowledge, a great deal of the cost and difficulty associated with heat treatment of metallic components can be reduced. In this paper, a combined numerical and analytical technique is developed to predict the stresses developed during heat treatment of a round shaft.

The problem of thermoelastoplastic and residual stress distribution in quenched bodies with material properties that were assumed to be temperature independent was first investigated by Sachs (Ref 1). He used the total plasticity theory to estimate the thermoelastoplastic stresses. Later Weiner and Huddleston (Ref 2) and Landu and Weiner (Ref 3) incorporated a simplified phase transformation model in their analysis. Later several others proposed different approaches for variety of transient thermoelastoplastic problems (Ref 4-12). Incorporation of the temperature dependency of material in the transient thermoelastoplastic problems using Mendelson's (Ref 13) method of successive elastic solution has been adopted by several investigators (Ref 13-23). Ishikawa (Ref 14, 15), Ishikawa and Hata (Ref 16), and Ishikawa et al. (Ref 17) solved the problem for a Ramberg-Osgood type of materials (Ref 18). Jahanian and Sabbaghian (Ref 19, 20) and Jahanian (Ref 21) extended the Mendelson's method for a material of linear strain hardening. Later Jahanian (Ref 22) used a hyperbolic sine law to investigate the metal plasticity in aluminum cylinder bars. Extension of Jian's metal plasticity model to a material with temperature dependent properties and incorporation of post yielding phenomenon (Ref 22), has been recently reported by Jahanian (Ref 23).

A simplified model for prediction of thermal stresses, which are developed during phase transformation of steel, has been addressed by Jahanian (Ref 24), Mitter et al. (Ref 25), Rammerstorfer et al. (Ref 26), and Ishikawa et al. (Ref 17).

This research considers the problem of transient thermal stresses in an infinitely long solid cylinder of low carbon steel with strain hardening and temperature dependent properties. The cylinder is rapidly cooled from the austenite temperature of 840 °C (Fig. 1). For thermoelastoplastic modeling, the constitutive equations developed by Jahanian (Ref 23) for a material with temperature dependent property are adopted. For phase transformation, two parts are considered: nucleation, according to Scheil's method, and phase growth according to a law developed by Johnson and Mehl (Ref 27). For martensitic transformation a law established by Koistenin and Murburger

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(Ref 28) is adopted. Non-additivity of pearlitic and bainitic nucleation suggested by Manning and Lorig (Ref 29) is taken into account by means of a correction factor to Scheil's summation at the transition from pearlitic to bainitic. Depending on the speed of cooling, martensite, pearlite, or bainite is formed in the specimen.

2. Theoretical Analysis

A low carbon solid cylinder of radius, a, as that described in Ref 30 is considered. At $t = 0$ this cylinder is assumed to be at uniform temperature, T_f, which is above the transformation temperature, A_3 At $t > 0$ the outside of the cylinder is instantaneously cooled to the room temperature, T_0. The analysis of the thermoelastoplastic stresses resulting from cooling of the cylinder are presented.

2.1 Fundamental Equation for Stress and Strain

Total strains developed during the cooling of the cylinder were the sum of strains due to elastic, thermal, and plastic deformation and the strain due to phase transformation:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \partial_{ij} \int \alpha dT + \varepsilon_{ij}^{p} + \varepsilon_{ij}^{ph} \qquad \text{(Eq 1)}$$

The following dimensionless parameters are defined as:

$$\sigma_r^* = \frac{(1-v)\sigma_r}{E_0\alpha_0(T_f - T_0)}$$

$$\varepsilon_r^* = \frac{(1-v)\varepsilon_r}{\alpha_0(T_f - T_0)}$$

$$B = B_1 \frac{E_0\alpha_0(T_f - T_0)}{(1-v)}$$

$$C = C_{11}/[(T_f - T_0) + 273] \qquad \text{(Eq 2)}$$

where the coefficient of thermal expansion, α, the elastic modulus, E, and the yield stress, σ_1, are assumed to be temperature dependent and have been defined as the product of two terms. The first term designated by a zero subscript is a dimensional part, and the second term designated by an asterisk is a dimensionless part, which is a function of temperature:

$$\alpha = \alpha_0\alpha^*(\theta) \quad E = E_0E^*(\theta) \quad \sigma_1 = \sigma_{10}\sigma^*(\theta) \qquad \text{(Eq 3)}$$

and

$$\alpha^* = 1 + \alpha_1\theta$$

$$E = 1 - E_1\theta^2$$

$$\sigma_1^* = 1 + \sigma_{11}\theta + \sigma_{12}\theta^2 + \sigma_{13}\theta^3 \qquad \text{(Eq 4)}$$

where α_1, E_1 σ_{11}, σ_{12}, and σ_{13} are the material constants.

Upon introducing the previous equations to the familiar stress strain relation and substituting the results into the equilibrium and strain compatibility equation, the following equation is obtained (Ref 23):

$$\begin{aligned} E^*\varepsilon_\theta^* = {} & \frac{1-2v}{2(1-v)} \int \varepsilon_\theta^* \frac{\partial E^*}{\partial \rho} d\rho + \frac{1}{2\rho^2(1-v)} \int \rho\varepsilon_\theta^* \frac{\partial E^*}{\partial \rho} d\rho \\ & + (1+v) \frac{1}{\rho^2} \int E^* \rho(\int \alpha^* d\theta) d\rho + \frac{1-2v}{2(1-v)} \int E^* (\varepsilon_r^{*p} \\ & + \varepsilon_r^{*ph} - \varepsilon_\theta^{*p} + \varepsilon_{\theta r}^{*ph}) \frac{d\rho}{\rho} + \frac{1-2v}{2\rho^2(1-v)} \int \rho E^* (\varepsilon_r^{*p} + \varepsilon_r^{*ph} \\ & - \varepsilon_\theta^{*p} + \varepsilon_\theta^{*ph}) d\rho - \frac{v}{\rho^2(1-v)} \int \rho E^* \varepsilon_z^* d\rho + C_1 + \frac{C_2}{\rho^2} \end{aligned} \qquad \text{(Eq 5)}$$

and

$$\begin{aligned} E^*\varepsilon_r^* = {} & \frac{1-2v}{1-v} \int \varepsilon_\theta^* \frac{\partial E^*}{\partial \rho} d\rho - E^*\varepsilon_\theta^* + (1+v)E^* \int \alpha^* d\theta \\ & + \frac{1-2v}{1-v} E^*(\varepsilon_r^{*p} + \varepsilon_r^{*ph}) + \frac{1-2v}{1-v} \int \frac{E^*}{\rho} (\varepsilon_r^{*p} + \varepsilon_r^{*ph} \\ & - (\varepsilon_\theta^{*\theta} + \varepsilon_\theta^{*ph}))d\rho - \frac{v}{1-v} E^*\varepsilon_z^* + 2c_1 \end{aligned} \qquad \text{(Eq 6)}$$

and the boundary conditions are:

$$\sigma_r^*(\rho = 0) \neq \infty \quad \sigma_r^*(\rho = 1) = 0 \qquad \text{(Eq 7)}$$

and

$$2\pi \int \sigma_z^* \rho d\rho = 0$$

Introducing Eq 7 to Eq 5 and 6, the constants of integration, C_1 and C_2, can be found:

$$C_2 = 0$$

$$\begin{aligned} C_1 = {} & -\frac{(1-2v)}{2(1-v)} \int_0^1 \varepsilon_\theta^* \frac{\partial E^*}{\partial \rho} d\rho - \int_0^1 \rho\varepsilon_\theta^* \frac{\partial E^*}{\partial \rho} d\rho \\ & + \int_0^1 E^*(\varepsilon_r^{*p} + \varepsilon_r^{*ph} - (\varepsilon_\theta^{*p} + \varepsilon_\theta^{*ph})) \frac{d\rho}{\rho} \\ & - (1-2v) \int_0^1 \rho E^* [\varepsilon_r^{*p} + \varepsilon_r^{*ph} + (\varepsilon_\theta^{*p} + \varepsilon_\theta^{*ph})] d\rho \\ & + 2v \int_0^1 \rho E^* \varepsilon_z^* d\rho - 2(1+v)(1-v) \int_0^1 \rho E^* (\int \alpha^* d\theta) d\rho] \end{aligned} \qquad \text{(Eq 8)}$$

$$\varepsilon_z^* = \left(\frac{1}{\int_0^1 E^* \rho d\rho}\right)\left\{(1-v)\int_0^1 E^*\left(\int \alpha^* d\theta\right)\rho d\rho - \int_0^1 \rho E^*[\varepsilon_\theta^{*p} + \varepsilon_\theta^{*ph} + (\varepsilon_r^{*p} + \varepsilon_r^{*ph})]d\rho\right\} \quad \text{(Eq 9)}$$

2.2 *Transient Temperature Distribution*

The transient temperature distribution in the cylinder can be found by solving the familiar Carslaw and Geagar's heat conduction equations (Ref 31).

$$\mathrm{div}(k(\mathrm{grad}T)) = \gamma C \frac{\partial T}{\partial t} \quad \text{(Eq 10)}$$

where k is the thermal conductivity, C is the specific heat, and γ is the mass density and are defined as:

$$k = k_0 k^*(\theta)$$
$$C = C_0 C^*(\theta)$$
$$\gamma = \gamma_0 \gamma^*(\theta) \quad \text{(Eq 11)}$$

where K^*, C^*, and γ^* are defined in Ref 32 and are not repeated here.

The solution of such a problem when the cylinder is cooled from the temperature, T_f, to the room temperature distribution can be obtained by using the familiar Carslaw and Jeager equation (Ref 31):

$$\theta(\rho,\tau) = \left(\frac{1}{k_1}\right)[1 - (1 - 2k_1\varphi)^{1/2}] \quad \text{(Eq 12)}$$

where:

$$\varphi = 1 - 2\beta_h\varphi_0 \sum_{m=1}^{\infty} e^{-\lambda_n^2 \tau} \frac{J_0(\lambda_n r)}{(\lambda_n^2 a^2 + \beta_h^2) J_0(\lambda_n a)} \quad \text{(Eq 13)}$$

where k_1 is a dimensionless parameter and defined in the following equation:

$$K^*(\theta) = 1 - K_1\theta \quad \text{(Eq 14)}$$

In the previous equations:

$$\varphi_0 = 1 - \frac{K_1}{2} \quad \beta_h = \frac{ah}{K_0} \quad s = \frac{K_0}{\gamma_0 C_0 b^2} t \quad \theta = \frac{T_f - T}{T_0} \quad \text{(Eq 15)}$$

where α_n can be found by solving the following equation:

$$\alpha_n J_1(\alpha_n) - \beta_h j_0(\alpha_n) = 0 \quad \text{(Eq 16)}$$

and β_h is the Biot number.

2.3 *Plastic Strain Increment*

For calculation of the plastic stress and strain the model developed by Jahanian (Ref 22, 23) is adopted. The details of the derivation of equations are given in Ref 23; however, the method is briefly described in this article. The plastic strains, which are parts of Eq 5 through 9, can be evaluated using the following equations:

$$\varepsilon^{*p}_{ij,(t=k)} = \varepsilon^{*p}_{ij,(t=k-1)} + d\varepsilon^{*p}_{ij,(t=k)} \quad \text{(Eq 17)}$$

It has been shown that (Ref 32):

$$d\varepsilon^p_{ij} = \left[\frac{mdS_{kl} - \alpha_{kl}dm}{m^2(S_{kl} - \alpha_{kl})} - \frac{d\sigma_y}{m\sigma_y}\right][S_{ij} - \alpha_{ij}] \quad \text{(Eq 18)}$$

where α_{ij} is the deviatoric back stress, which can be related to the plastic strain through:

$$\alpha_{ij} = m(\theta,\tau)\varepsilon^p_{ij} \quad \text{(Eq 19)}$$

and:

$$S_{ij} = \alpha_{ij} - \left(\frac{1}{3}\right)\sigma_{kk}\delta_{kk} \quad \text{(Eq 20)}$$

To show the dimensionless form of Eq 18, the following is defined:

$$\sigma_e = \sqrt{\frac{3}{2}(S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij})} \quad \text{(Eq 21)}$$

$$\varepsilon^p = \sqrt{\frac{2}{3}\varepsilon^p_{ij}\varepsilon^p_{ij}} \quad \text{(Eq 22)}$$

$$\alpha_e = \sqrt{\frac{3}{2}\alpha_{ij}\alpha_{ij}} \quad \text{(Eq 23)}$$

$$d\varepsilon^p = \sqrt{\frac{2}{3}d\varepsilon^p_{ij}\,d\varepsilon^p_{ij}} \quad \text{(Eq 24)}$$

The following dimensionless parameters are introduced to modify the previously mentioned equations:

$$\sigma^*_{v1} = \frac{(1-v)\sigma_y}{E_0\alpha_0T_0} - \frac{(1-v)\sigma_y}{E_0\alpha_0T_0}(1-n)(0.02)$$

$$\sigma^*_e = \frac{(1-v)\sigma_e}{E_0\alpha_0T_0} \quad \alpha^*_e = \frac{(1-v)\alpha_e}{E_0\alpha_0T_0}$$

$$\varepsilon^*_p = \frac{(1-v)\varepsilon^*_p}{\alpha_0T_0} \quad m^* = \frac{(1-v)m}{E_0\alpha_0T_0} \quad S^*_{ij} = \frac{(1-v)S_{ij}}{E_0\alpha_0T_0} \quad \text{(Eq 25)}$$

It is shown that (Ref 32):

$$\alpha_{\mathrm{e}}^{*} = \frac{2}{3} m{*}\varepsilon^{*\mathrm{p}}$$

$$d\varepsilon_{\mathrm{ij}}^{*\mathrm{p}} = \frac{3}{2} \frac{d\varepsilon^{*\mathrm{p}}}{\sigma_{\mathrm{e}}^{*}} (S_{\mathrm{ij}}^{*} - \alpha_{\mathrm{ij}}^{*}) \qquad \text{(Eq 26)}$$

Upon further simplification:

$$d\varepsilon_{\mathrm{ij}}^{*\mathrm{p}} = \left[\frac{m{*}dS_{kl}^{*} - \alpha_{kl}^{*}\, dm^{*}}{m^{*2}(S_{kl}^{*} - \alpha_{kl}^{*})} - \frac{d\sigma_{y1}^{*}}{m^{*}\sigma_{y1}^{*}} \right] (S_{\mathrm{ij}}^{*} - \alpha_{\mathrm{ij}}^{*}) \qquad \text{(Eq 18a)}$$

Metallurgical Response. Consider a solid cylinder that is heated to a temperature above A_3 and then kept at that temperature until isothermal temperature is reached. Accordingly the cylinder becomes stress free at that temperature. The cylinder is then rapidly cooled, and during cooling, pearlite, bainite, and martensite transformations occur. The phase transformations calculation is composed of nucleation and phase growth during pearlitic and bainitic transformation, as well as martensitic transformation.

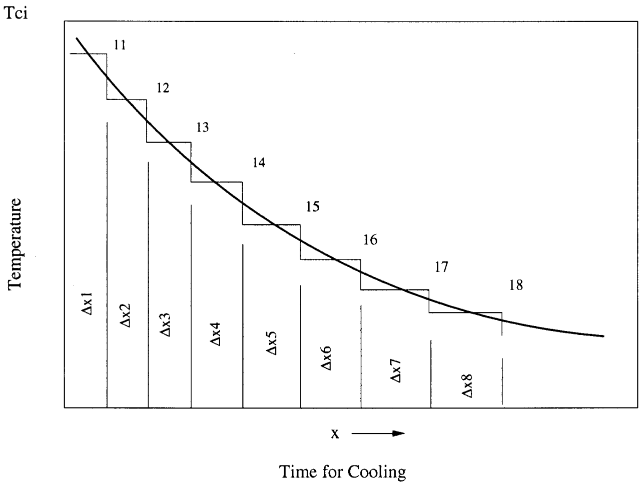

Fig. 1 Graphical representation of Eq 27

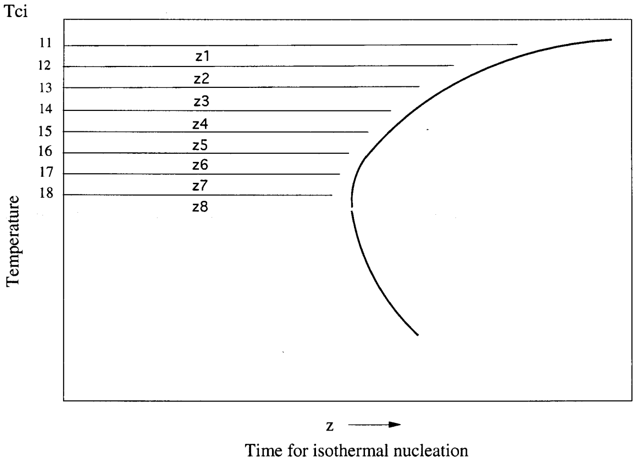

Fig. 2 Graphical representation of Eq 28

Nucleation. Prediction of the onset of nucleation has been addressed by several researchers. In this research the additivity rule of Manning and Lorig (Ref 29) is adopted. This technique relates the initial phase transformation time during continuous cooling and the initial phase transformation time at a constant temperature until the onset of nucleation. Based on this law, the time spent at a particular temperature divided by the time required for beginning of the transformation at that temperature is considered to represent a fraction of the total nucleation time required. The transformation begins when the summation of this fraction divided by the rate of cooling is equal to unity:

$$\text{I.P.}(T_n) = \frac{dt_1}{Z_1} + \frac{dt_2}{Z_2} + \ldots \frac{dt_n}{Z_n} = \sum_{t=0}^{t=t_a} \frac{dt(T)}{Z(t)} \qquad \text{(Eq 27)}$$

where I.P. is the incubatic period. As t approaches n, each term becomes increasingly smaller.

$$\text{I.P.}(T_n) = \int_{t=0}^{t=t_n} \frac{dt}{Z(T)} = \int_{A_1}^{T_n} \frac{dt/T}{Z(T)}\, dT \qquad \text{(Eq 28)}$$

where A_1 is the equilibrium transformation temperature. When the rate of cooling is constant Eq 28 becomes:

$$\text{I.P.}(T_n) = \frac{1}{\alpha} \int_{A}^{T_n} \frac{dT}{Z(T)} \qquad \text{(Eq 29)}$$

At the temperature T_n, for which I.P. = 1, the phase transformation begins. Figures 1 and 2 show the graphical representation of the above equation.

Phase Growth. The reaction developed by Johnson and Mehl (Ref 27) and Avarmi (Ref 33-35) is used for phase growth. A law established by Koistinen and Marburger (Ref 28) is adopted for martensite transformation. Based on Johnson and Mehl (Ref 27) and Avarmi (Ref 33-35), the following relation holds:

$$y = 1 - \exp(-bt^{n}) \qquad \text{(Eq 30)}$$

where b and n can be found by using two arbitrary points on the isothermal temperature transformation (ITT) curve. The percentage of bainite and pearlite transformation at each time step can be evaluated using the technique described in the numerical procedure section. In the Eq 30 relation, y is the fraction of the phase formed by the transformation. Finally the strain due to phase transformation can be evaluated using the following relation (Ref 28):

$$\varepsilon^{\mathrm{tp}}(\sigma, z) = k\sigma_{\mathrm{e}}\mathrm{f}(z) \qquad \text{(Eq 31)}$$

where:

$$f(z) = y \quad k = 10^{-4}\,\mathrm{MPa}^{-1}$$

and σ_e is the equivalent stress (Ref 21).

Martensitic Transformation. The kinetics of martensite transformation have been studied by Koistinen and Marburger (Ref 28) who proposed the following relation:

$$y=1-exp[-ea(Ms-T)]$$

where for most steel:

$$\alpha = 1.1 \times 10^{-2} k^{-1}$$

$$M_s = \text{Beginning of Martensite Transformation}$$

Numerical Procedure. For the numerical analysis the cross section of the cylinder was initially partitioned into 100 unequal elements in the radial direction. The time increment initially consisted of 50 unequal intervals. Sections radially near to the outer surface and the time increment in the early cooling stages were smaller than at interior sections and:

a) if $|(\Delta\varepsilon th)_c| > |(\Delta\varepsilon^{ph})_c|$ and $|(\Delta\varepsilon^{th})_0| > |(\Delta\varepsilon^{th})_0|$

then

$|(\Delta\varepsilon^{th} + \Delta\varepsilon^{ph})_c| > |(\Delta\varepsilon^{th} + (\Delta\varepsilon^{ph})_c)_0|$

b) if $|(\Delta\varepsilon^{th})_c| > |(\Delta\varepsilon^{ph})_c|$ and $|(\Delta\varepsilon^{th})_0| < |(\Delta\varepsilon^{ph})_0|$

then

$|(\Delta\varepsilon^{th} + \Delta\varepsilon^{ph})_c| < |(\Delta\varepsilon^{th} + (\Delta\varepsilon^{ph})_c)_0|$

c) if $|(\Delta\varepsilon^{th})_c| > |(\Delta\varepsilon^{ph})_c|$ and $|(\Delta\varepsilon^{th})_0| < |(\Delta\varepsilon^{ph})_0|$

then the following conditions must be met

$(\Delta\varepsilon^{e+p})_c = (\Delta\varepsilon)_c - (\Delta\varepsilon^{th} + (\Delta\varepsilon^{ph})_c > 0$

and

$(\Delta\varepsilon^{e+p})_0 = (\Delta\varepsilon)_0 - (\Delta\varepsilon^{th} + \Delta\varepsilon^{ph})_0 > 0$

At each time step the previous conditions were checked. If the conditions were satisfied, the results were accepted. Otherwise a smaller time increment was selected, and the same procedure was repeated.

Thermoelastoplastic Strains. For thermoelastoplastic strains, the method used in Ref 23 is used, and it is not repeated here.

Results of Numerical Calculation. For the purpose of discussion and comparison with experimental results, the following data for steel of 0.3% C, 0.2% Si, 0.45% Mn, 1.3% Cr, and 4.45% Ni is used (Ref 24):

$$k_0 = 59.7 w/mk$$

$$E_0 = 206.0 \times 10^9 \text{ N/m}^2$$

$$E_1 = 2.34 \times 10^{-3} \lambda^2$$

$$\sigma_{10} = 295 \times 10^6 \text{ N/m}^2$$

$$\sigma_{12} = -5.5 \times 10^{-3} \lambda^2$$

$$\sigma_{11} = 0.017\lambda$$

$$T_f = 840 \text{ °C}$$

$$k_1 = 0.0334\lambda$$

$$\sigma_{13} = 2.53 \times 10^{-2} \lambda$$

$$T_0 = 0.0 \text{ °C}$$

where λ is the loading parameter and is defined as:

$$\lambda = \frac{E_0 \alpha_0 T_0}{(1 - v)\sigma_0}$$

Coefficient of Thermal Expansion. The coefficient of thermal expansion changes with temperature (Ref 16, 17, 19-21). However this change during phase transformation is far greater than the change in normal temperatures. The following data from Ref 16, 17, 19 to 21, and 30 has been used.

For the portion of the cylinder that undergoes martensite transformation:

$$\alpha_0 = 1.12 \times 10^{-8} C^{-1} \text{ at } 0.605 > \theta \geq 0$$

$$\alpha_0 = 1.97 \times 10^{-5} C^{-1} \text{ at } 0.605 < \theta \leq 0$$

and the portion for which there is a pearlite or bainite transformation:

$$\alpha^*(\theta) = 1.34 \text{ at } 1 \geq \theta > 0.988$$

$$\alpha^*(\theta) = -2.19 \text{ at } 0.988 \geq \theta \geq 0.866$$

$$\alpha^*(\theta) = 1.0 \text{ at } 0.866 \geq \theta \geq 0$$

3. Discussion of Results

It is known that during cooling of low carbon steel when the speed of cooling is high enough, the formation of pearlite and bainite is avoided and martensite is produced. During this

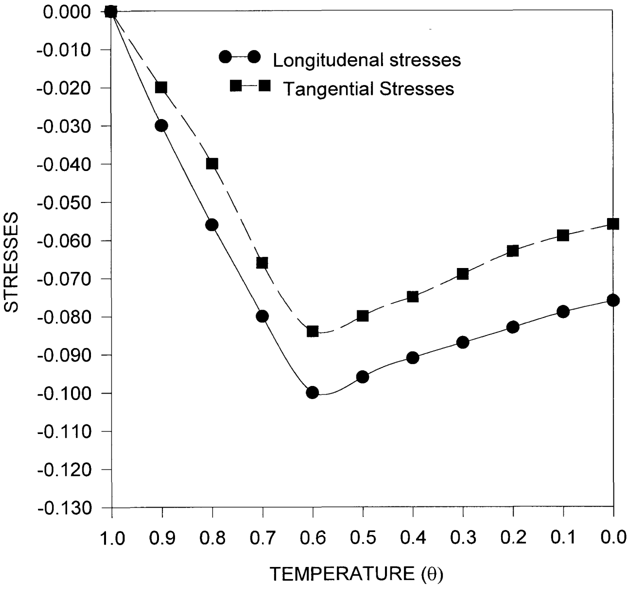

Fig. 3 Thermoelastoplastic stresses at the surface of the cylinder during cooling

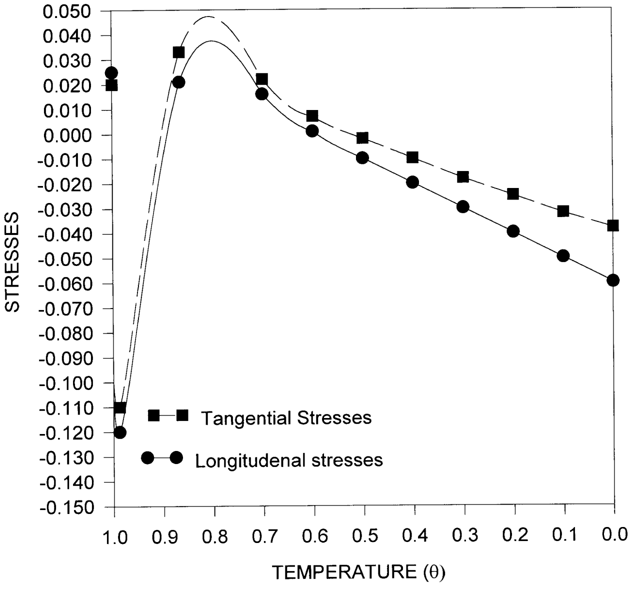

Fig. 4 Thermoelastoplastic stresses at the center of the cylinder during cooling

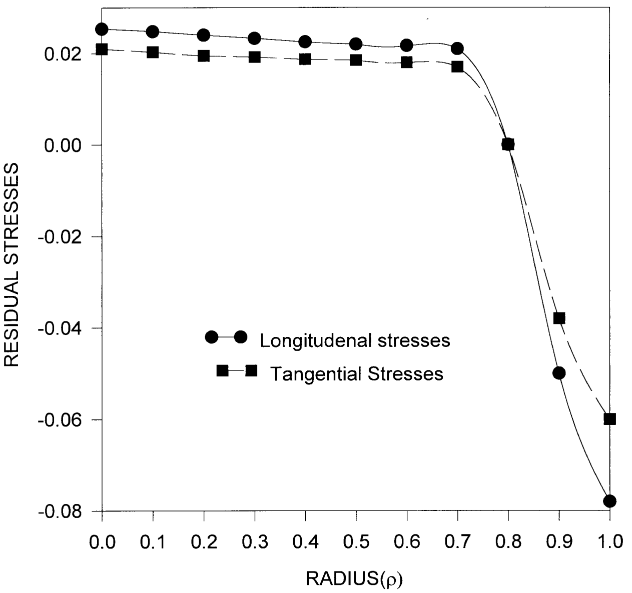

Fig. 5 Residual stress distribution

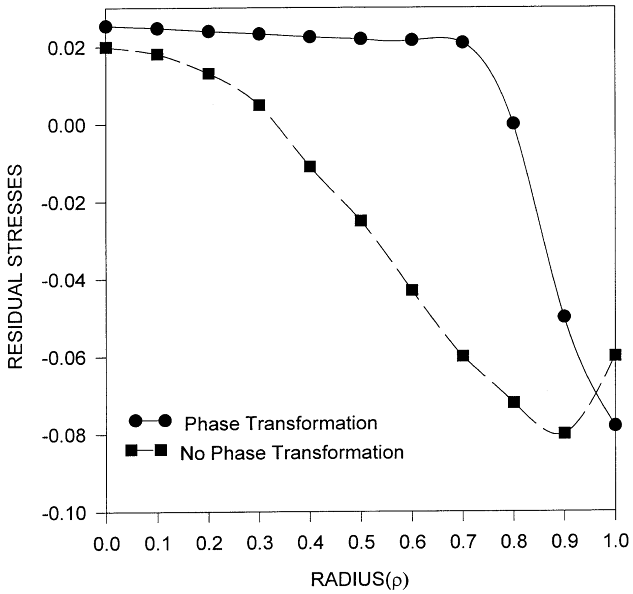

Fig. 6 Effect of phase transformation on accuracy of results

process austenite is completely transformed to martensite only at the surface of the cylinder. Figure 3 shows the tangential and longitudinal stresses at the surface of the cylinder during rapid cooling. A close examination of these figures reveals that compressive stresses have been developed up to a temperature of $\theta = 0.605$, after which tensile stresses are developed. This is visible in the decreasing trend up to $\theta = 0.605$, at which point the stress suddenly follows an increasing trend (the development of tensile stresses). This result occurs because, as the cylinder is rapidly cooled, the formation of pearlite and bainite is avoided at the surface of the cylinder (due to the high rate of cooling). For this reason, the trend of curve temperatures of 723 and 830 °C, which is equivalent to $\theta = 0.866$ and $\theta = 0.988$, did not change. On the contrary, Fig. 4 depicts that the trend of curve changes at $\theta = 0.866$ and $\theta = 0.988$, which are equivalent to 723 and 830 °C. This is due to the fact that at the center of the cylinder, formation of pearlite and bainite occurs.

At $\theta = 0.605$, martensite transformation occurs at the surface and the stresses increase. During cooling, the cylinder contracts uniformly; however, once martensite transformation occurs, it begins to expand. Figure 3 depicts this result. Figure 5 shows the residual stresses across the radius of cylinder. The magnitude of these stresses at the vicinity of the surface are higher than at the center. Figure 4 shows the thermoelastoplastic stresses at the center of the cylinder. Figure 6 depicts the effect of phase transformation on the accuracy of the results.

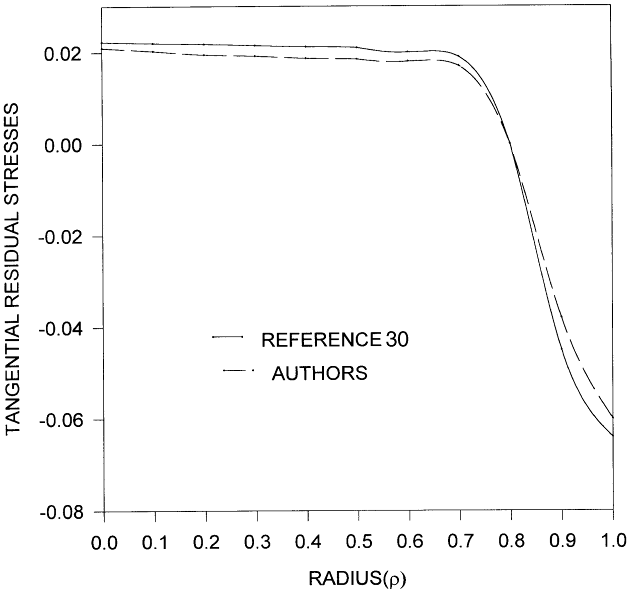

Fig. 7 Comparison of the results with experimental data

Phase transformation is an important factor to the considered during numerical simulation. When the phase transformation is neglected, the results are underestimated. Figure 7 shows the authors' results against the experimental results of Ref 30. Figure 7 shows a good numerical agreement within 95% of experimental.

4. Conclusions

The stresses developed in a solid cylinder during quenching were evaluated, and the following conclusions were derived from the results:

- Phase transformation is an important factor in evaluating residual stresses during induction hardening.
- The level of beneficial residual stresses at the surface are considerably higher than the tensile residual stresses at the center of the cylinder. This is due to formation of martensite at the surface and pearlite at the center.

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